

Single-slit diffraction plots

by Sohrab Ismail-Beigi, February 5, 2009

On the following pages are a set of computed single-slit intensities. The slit is long and narrow (i.e not circular but linear). The distance to the screen is $L=1$ m. I fixed the wavelength to be $\lambda=500$ nm as a typical optical wavelength for light. The qualitative features are not dependent on this. The variable is the slit width a which is varied from small to large values.

As discussed in the book and in lecture, for small a we have single-slit diffraction; for large a , we should have ray physics and a beam of light. The point of this exercise is to see how we transition smoothly between the two as a function of a . Actually, only ratios of distances matter so we could have fixed a and L and changed λ instead.

On each page, the intensity is plotted (vertical) as a function of the position (y) away from the center of the screen ($y = 0$) which is the closest point to the slit. The intensity is plotted in blue. Vertical red lines mark the position of the slit edges at $-a/2$ and $a/2$ in order to enable comparison of the size of the features on the screen to the size of the ideal beam image of the slit. The units of the intensity are arbitrary.

To remind you, the standard diffraction theory in the Fraunhofer (far-field, $a \ll L$) limit says that the angles for the dark fringes (zero intensity points) are given by

$$\theta_{dark} = \sin^{-1} \left(\frac{p\lambda}{a} \right) \quad \text{for } p = \pm 1, \pm 2, \pm 3, \dots$$

In the small angle limit where $\lambda/a \ll 1$, which will be true for all the plots, we have $\theta \approx p\lambda/a$. The position of the dark fringe on the screen is then

$$y_{dark} = L \tan(\theta_{dark}) \approx L\theta_{dark} = L \cdot \frac{p\lambda}{a}.$$

The key quantity determining whether wave diffraction is the proper view or a ray model is more appropriate is whether the quantity $a^2/(L\lambda)$ is smaller or larger than roughly one, respectively.

For those interested in the math, the intensity at some point y is given by the integrating the phasors for Huygens' wavelet sources over the entire slit (the variable u)

$$I(y) = \left| \int_{-a/2}^{a/2} du \frac{\exp(2\pi i r / \lambda)}{r} \right|^2 \quad \text{where } r = \sqrt{L^2 + (y - u)^2}.$$

The integral is done numerically to high precision for each plot for a dense grid of y values.

In the classic small angle limit where $|u| < a/2 \ll L$, we have with $\tan(\theta) = y/L$

$$r \approx \sqrt{L^2 + y^2} - yu/\sqrt{L^2 + y^2} = L/\cos(\theta) - u\sin(\theta)$$

and the integral can be done analytically to give

$$I(y) \approx \frac{a^2}{L^2 + y^2} \cdot \frac{\sin(\pi a \sin(\theta)/\lambda)}{\pi a \sin(\theta)/\lambda}.$$

Single-slit diffraction: $L=1$ m, $\lambda=500$ nm, $a=5 \times 10^{-5}$ m

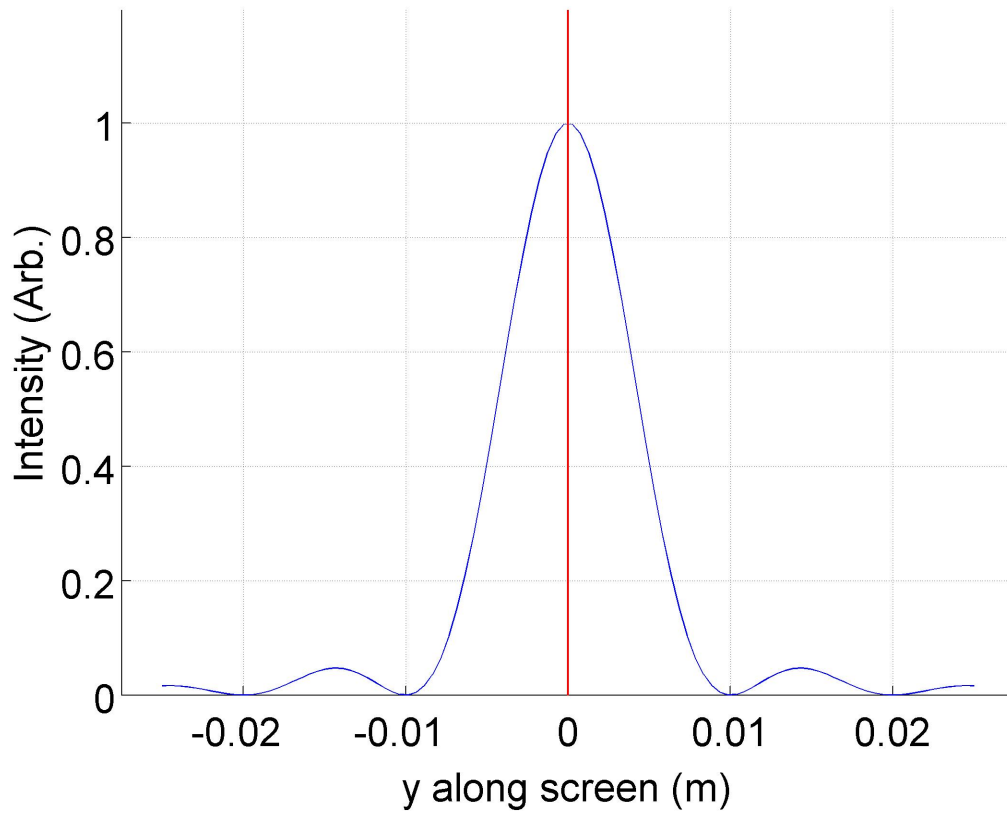


Figure 1: This is a very narrow slit where $a = 5 \times 10^{-5}$ m wide. Furthermore, $\lambda/a = 0.01 \ll 1$ so we are in the small angle limit for sure. As expected, we see the classic diffraction intensity pattern from a single-slit. The first dark spot is at $y = 0.01$ m just as it should be from the formula. The scale of the horizontal (y) axis is some 1000 times larger than the slit size, so the slit looks like just a dot at $y = 0$: it is invisible on this scale. Here $a^2/(L\lambda) = 0.005$ and we are safely in the diffraction picture.

Single-slit diffraction: $L=1$ m, $\lambda=500$ nm, $a=1 \times 10^{-4}$ m

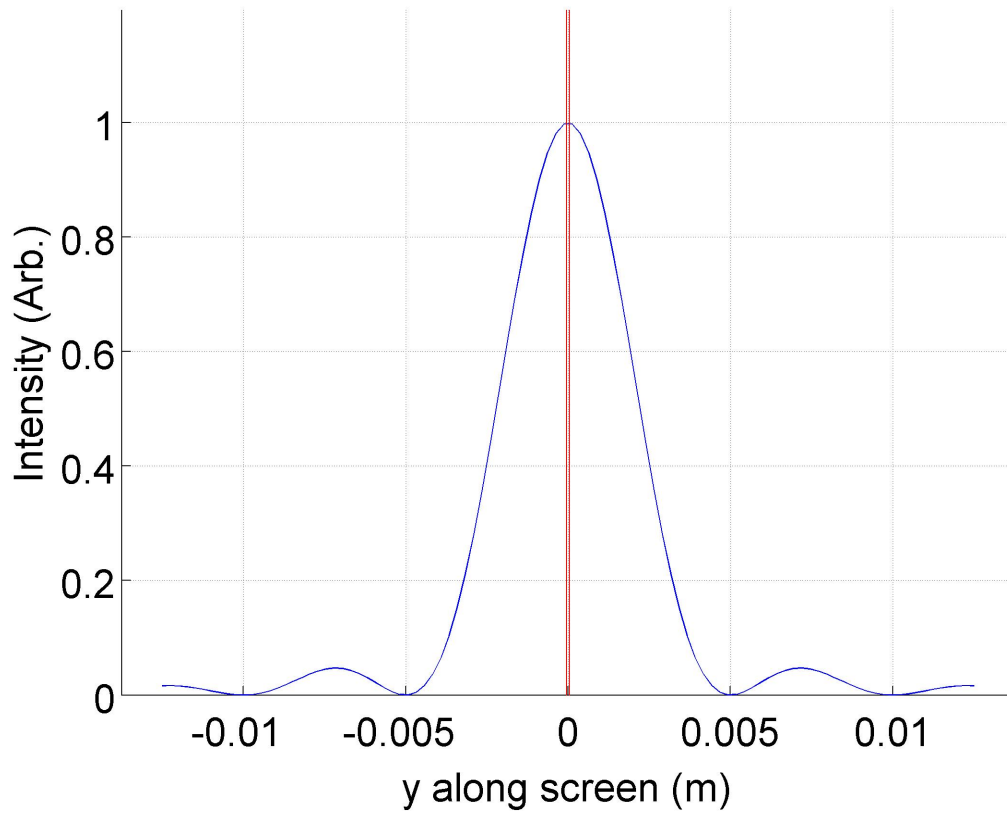


Figure 2: Now $a = 10^{-4}$ m. Things look just like before but of course the scale of the horizontal axis has shrunk by a factor of two. But if you look very carefully, you can see a very small spacing between the two vertical lines: the slit is still small on this scale but not invisible. Here $a^2/(L\lambda) = 0.02$.

Single-slit diffraction: $L=1$ m, $\lambda=500$ nm, $a=5 \times 10^{-4}$ m

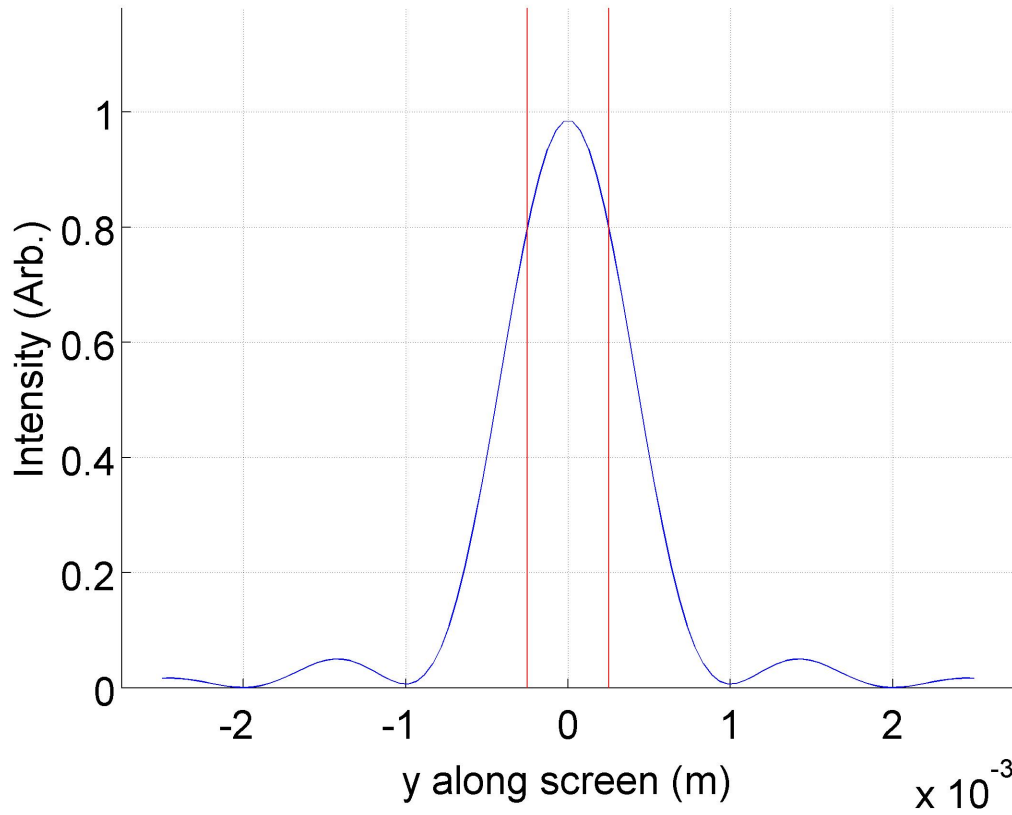


Figure 3: Here $a = 5 \times 10^{-4}$ m wide. The slit image is quite easily visible and is almost as wide as the central bright diffraction fringe. The overall intensity pattern, however, is like before. But note that at what should be the dark spot at $y=10^{-3}$ m, the intensity is small but non-zero: we are beginning to see small departures from the standard diffraction formula. Here $a^2/(L\lambda) = 0.5$ which is getting close to unity.

Single-slit diffraction: $L=1$ m, $\lambda=500$ nm, $a=1 \times 10^{-3}$ m

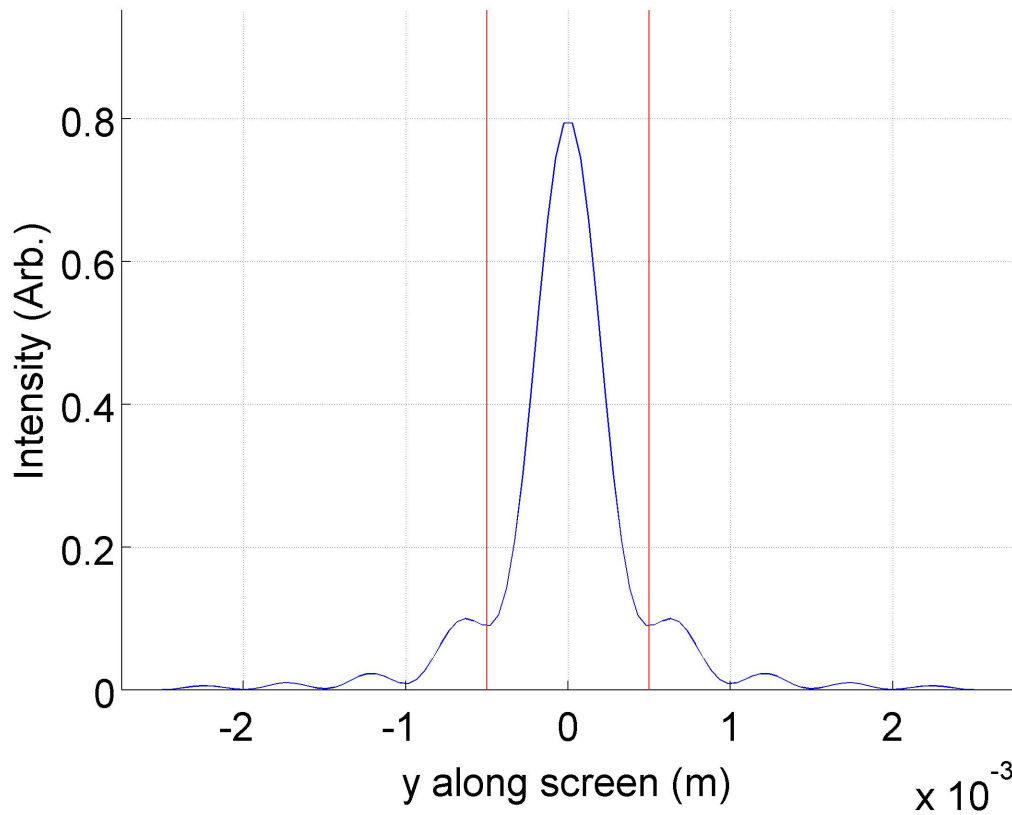


Figure 4: Here $a = 10^{-3}$ m = 1 mm wide. The book uses this slit size as a typical division point between the diffraction and ray pictures. The slit size has overtaken the size of the central “diffraction” fringe (in actuality there is no fringe just a bump). The shape of the intensity is qualitatively similar to the previous plots only in that there is a tall central bump and small side bumps, but we see no dark spots in the expected places. Now $a^2/(L\lambda) = 2$ and the diffraction picture is starting to break down. But note that this is not a sharp division: the ray picture is not quite valid yet as there is a good deal of intensity outside the slit positions.

Single-slit diffraction: $L=1$ m, $\lambda=500$ nm, $a=5 \times 10^{-3}$ m

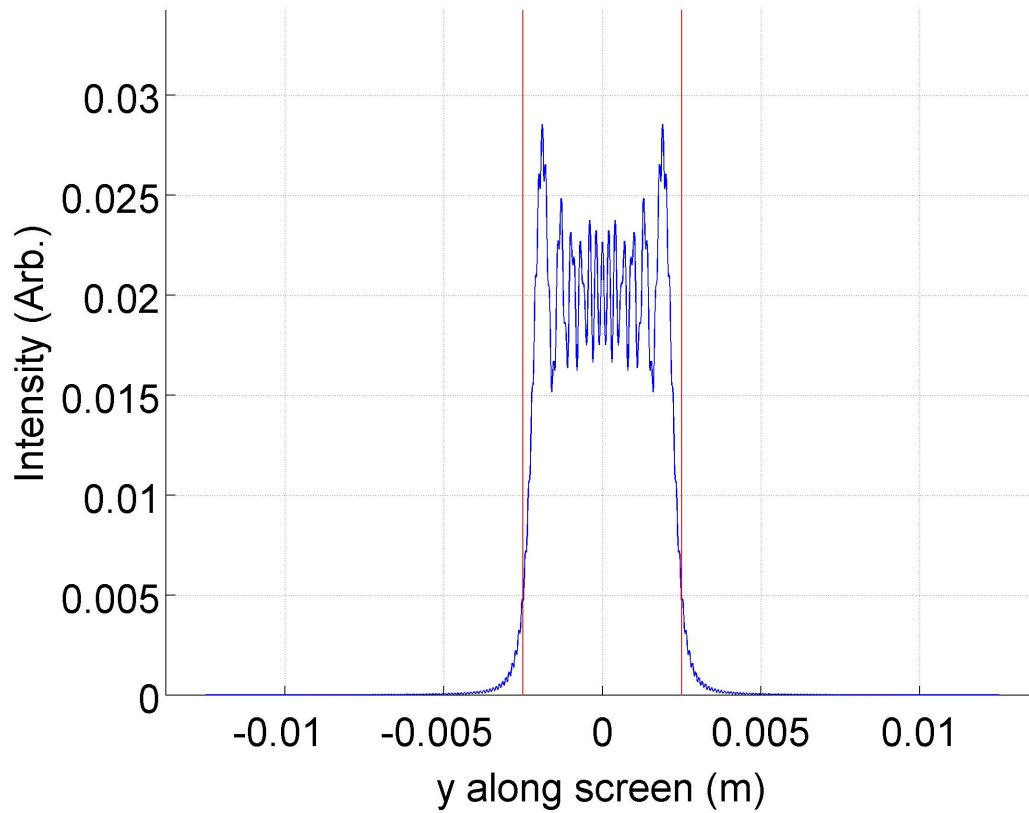


Figure 5: Here $a = 5 \times 10^{-3}$ m. Now $a^2/(L\lambda) = 50$ and the diffraction picture is essentially broken down completely; the intensity looks qualitatively different. There is a region sandwiched between the positions of the slit edges where the intensity is high and roughly uniform, and outside it is quite small and rapidly decaying to zero. And yet the wave nature is still present: the intensity fluctuates rapidly along y . These wiggles come from detailed interference conditions coming from adding up the sources along the entire slit.

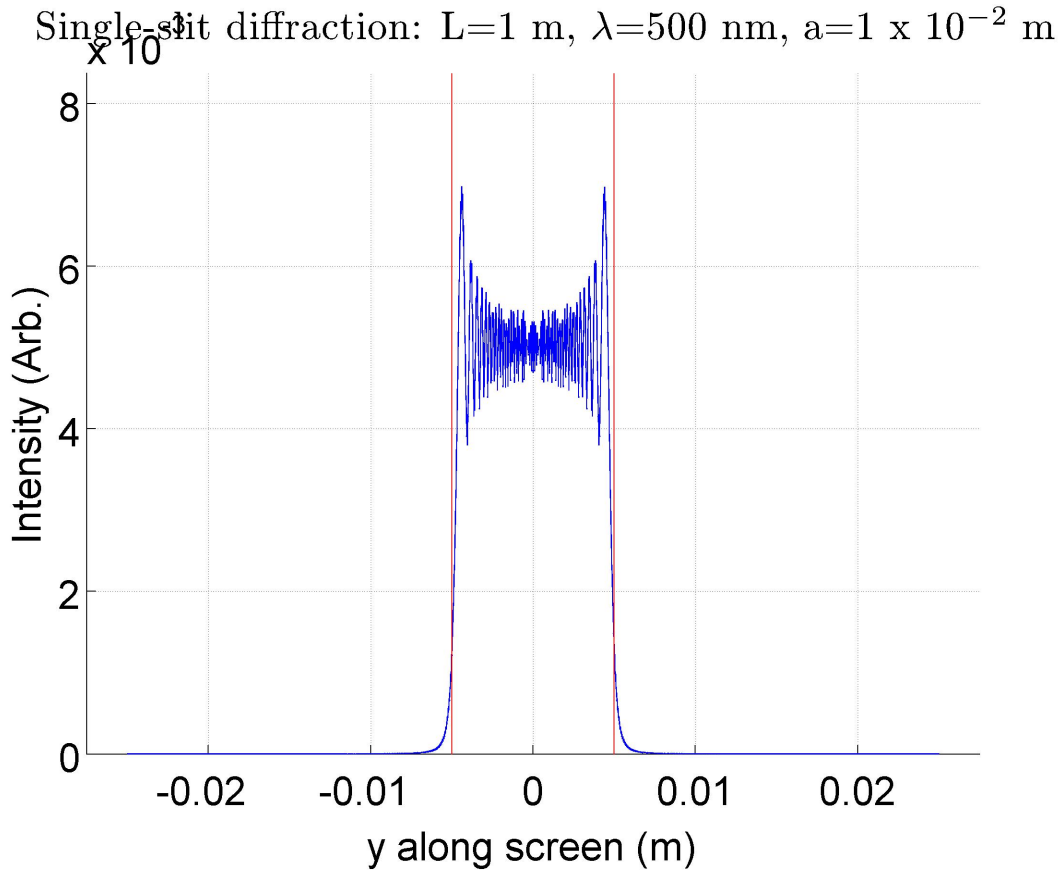


Figure 6: Here $a = 10^{-2}$ m = 1 cm. Now $a^2/(L\lambda) = 200$. The region between the slit edges has essentially all the intensity. Ignoring for the moment the rapid wiggles, the intensity is constant between the slit edges as it would be for a beam of light coming straight out of the slit itself: the ray picture has emerged. The little wiggles are due to detailed interference and show that this is all actually coming from the wave nature in the end. However, the horizontal scale is now in cm and a human eye could not resolve the tiny and rapid wiggles in the intensity: the eye would smooth over them and the intensity would look smooth and constant in the bright region. However, if one could measure the intensity precisely and on a short length scale, one would be able to see all those wiggles and would be forced to conclude that the ray picture is not complete and there is some “waving” going on somewhere.