Variable range hopping

Simple derivation based on what is on Wikipedia

We assume electronic states are localized in space with exponential decay length λ. $\alpha \equiv 1/\lambda$

They are randomly distributed in space up to R_{max}.

Their energies are randomly distributed over energy bandwidth B. Total states N_{states}.

Question: an $\bar{e}$ is in state at $r=0$ with zero energy what is probability for it to tunnel at finite temperature to a nearby site to get transport?

Prob. to go to a state localized at $\vec{r}$ with energy $\mathcal{E}$ is $p \sim \exp(-2\alpha r - \mathcal{E}/T)$ $(k_B = 1)$

Key quantity is $\mathcal{R} \equiv 2\alpha r + \mathcal{E}/T$

& $p \sim \exp(-\mathcal{R})$

Next, lets ask how many states $N(\mathcal{R})$ have $\mathcal{R}$ values between 0 & $\mathcal{R}$?

The density of states $D(r, \mathcal{E})$ of states at $(r, \mathcal{E})$ is by our assumptions

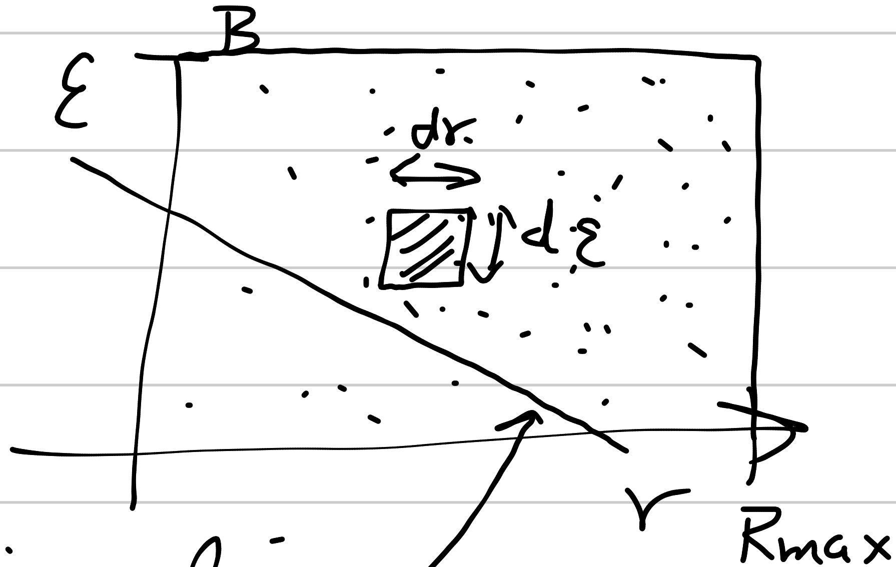
in d dimensions $D(r,\mathcal{E}) = c\cdot r^{d-1}$ $\quad c = ?$

$$N_{states} = \int_0^{R_{max}} dr \int_0^{B} d\mathcal{E}\, D(r,\mathcal{E}) = c\cdot \frac{R_{max}^{d}}{d}\cdot B$$

so $c = N_{states}\cdot \frac{d}{B\cdot R_{max}^{d}}$

Diagram: each dot is a state, N_{states} total, # in box is

$D(r,\mathcal{E})\, dr\, d\mathcal{E}$



$\mathcal{R}$ = fixed = $2\alpha r + \mathcal{E}/T$ is a line

we want $\iint^{line} d\mathcal{E}\, dr\, D(r,\mathcal{E}) = \mathcal{N}(\mathcal{R})$

$$\mathcal{N}(\mathcal{R}) = \int_0^{\mathcal{R}/2\alpha} dr \int_0^{\mathcal{R}T - 2\alpha r T} d\mathcal{E}\cdot c\cdot r^{d-1}$$

$$= cT\cdot \int_0^{\mathcal{R}/2\alpha} dr\cdot r^{d-1}(\mathcal{R} - 2\alpha r)$$

$$= c\cdot T/(2\alpha)^d \int_0^{\mathcal{R}} du\, u^{d-1}(\mathcal{R} - u)$$

$$= \frac{cT}{(2\alpha)^d}\,\frac{\mathcal{R}^{d+1}}{d(d+1)} = N_{states}\cdot\frac{T}{B}\cdot\frac{\mathcal{R}^{d+1}}{(2\alpha R_{max})^d}\cdot\frac{1}{(d+1)}$$

$$\propto T\mathcal{R}^{d+1}$$

Finally, the most probable transition is to the state with smallest $\mathcal{R}$.
So we must start at $\mathcal{R}=0$ and increase $\mathcal{R}$ until $N(\mathcal{R}) \sim 1$ which is $\frac{1}{g} \cdot T \cdot \mathcal{R}^{d+1} \sim 1$ & $g = \frac{(2\alpha R_{max})^d \cdot B \cdot (d+1)}{N_{states}}$

or $\mathcal{R} \sim \frac{\text{const.}}{T^{1/(d+1)}}$

So final prob. of hop is $\sim \exp\left(-\left(\frac{g}{T}\right)^{\frac{1}{d+1}}\right)$

e.g. $d=3 \sim \exp\left(-\frac{g^{1/4}}{T^{1/4}}\right)$