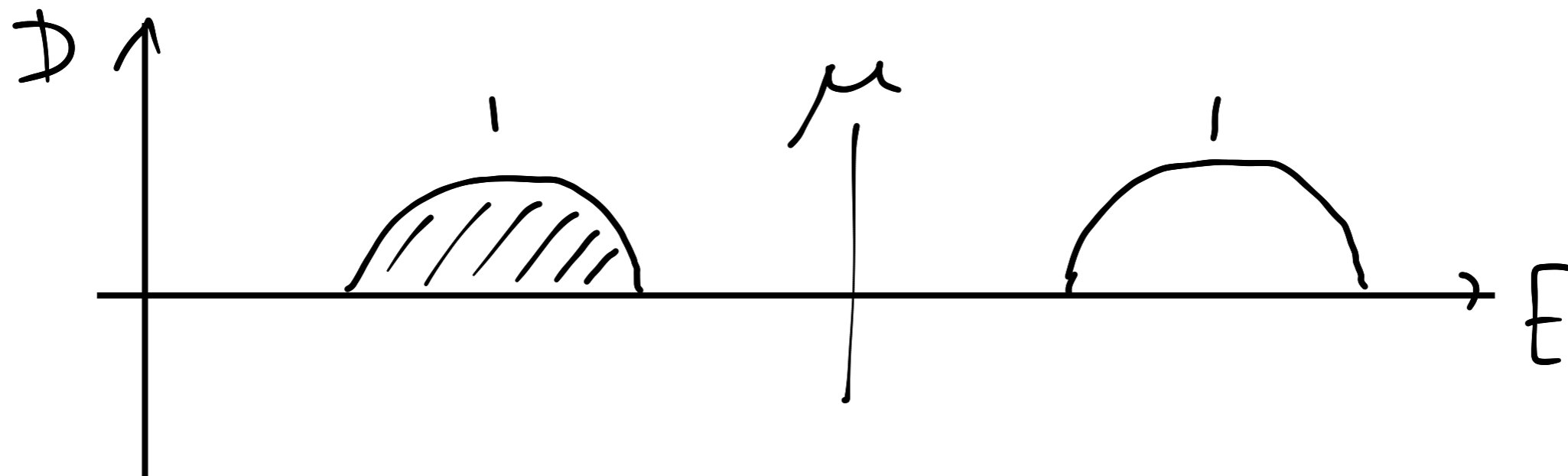
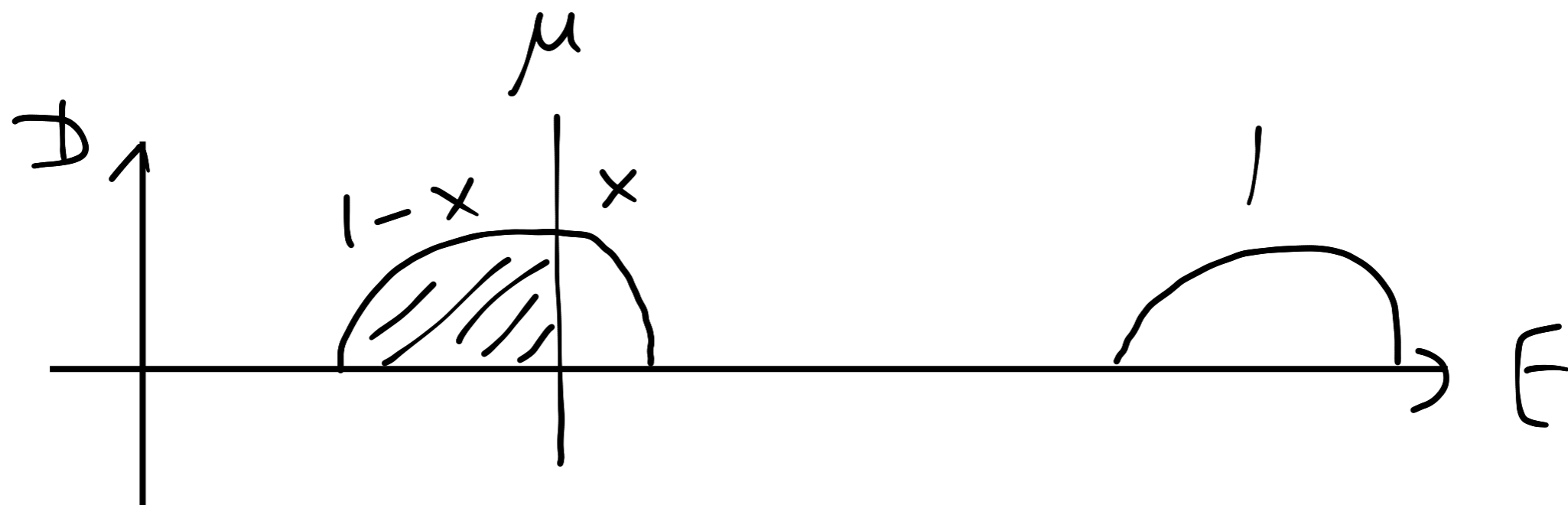


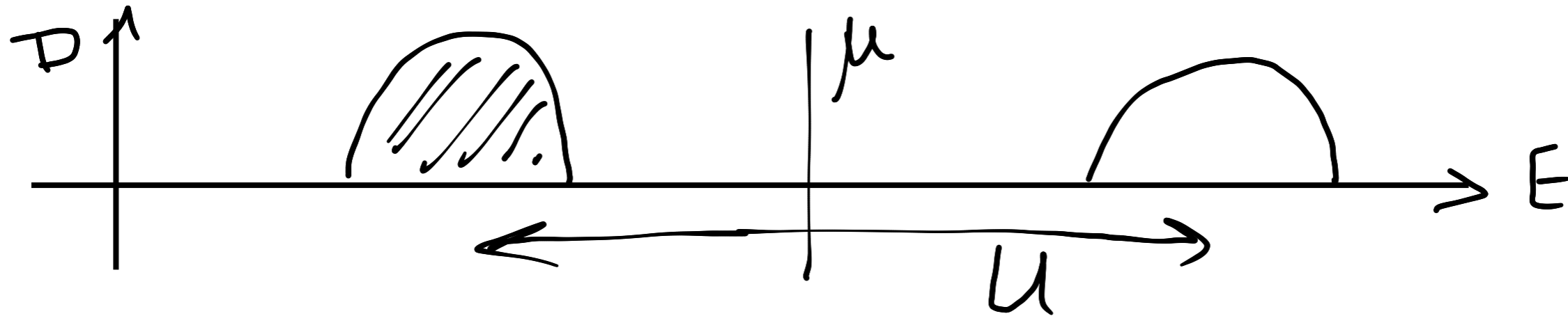
A band insulator with 1 filled and 1 empty band per site.



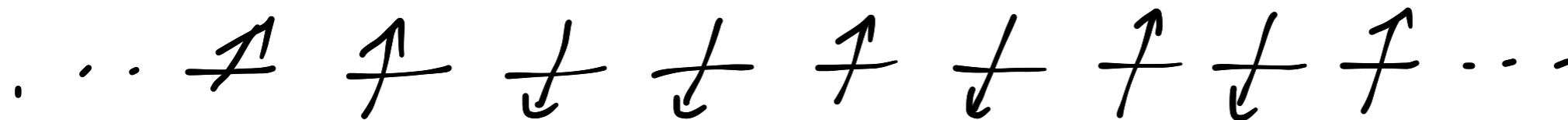
Hole dope it with fraction  $x$ . Fermi level moves down into valence band. Fraction  $1-x$  is filled and fraction  $x$  is emptied. So we get  $x$  empty states. All as expected.



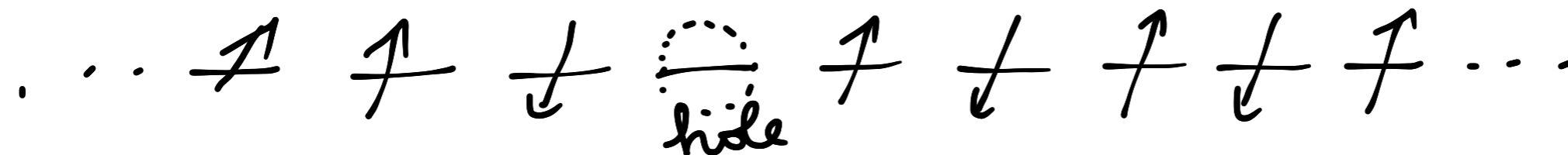
Now consider a 1 orbital per site Mott system with 1 electron per site. In limit  $U \gg t$  we get very narrow Hubbard bands (filled lower band, empty upper band). Things look similar at this level to band insulator.



Big difference is nature of electronic states. They are spatially localized. So for whatever spin configuration we might have, we have one electron per site localized. Pictorially something like this for ground state

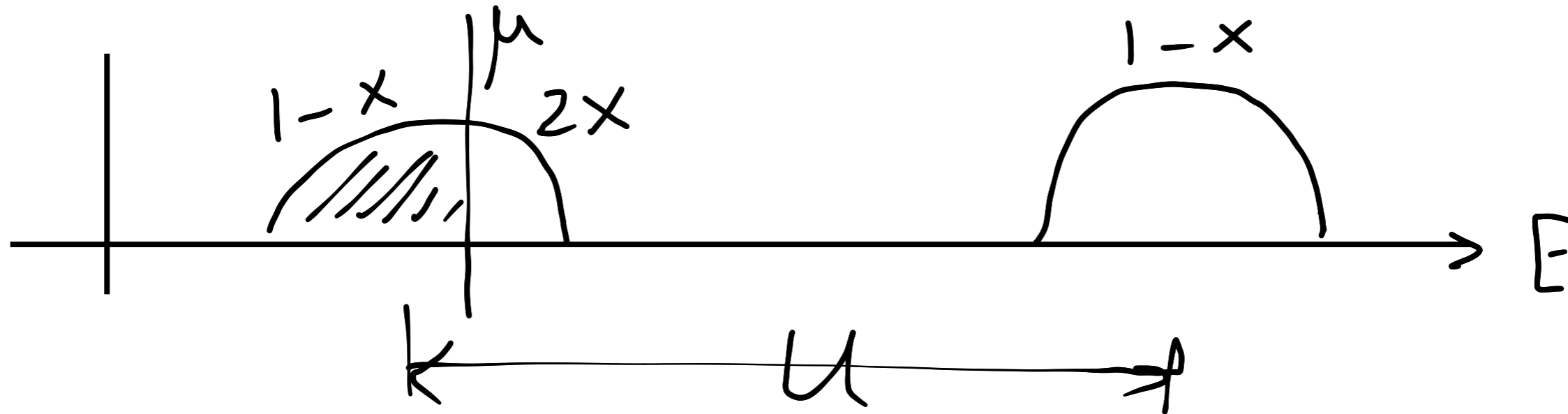


Now hole dope this at fraction  $x$ . So we remove a fraction  $x$  of electrons (let  $x$  be small). So the ground state configuration of doped system might look like



Now we ask what is the appropriate density of states (i.e. spectral function) for this ground state. If we want to remove an electron (occupied states below Fermi level) we have  $1-x$  choices of electrons to remove as there are  $1-x$  sites left with electrons in them. But if we want to add electrons (unoccupied states above Fermi level) there are two distinct ways with different energies: we could add an up or down spin electron to empty hole site (two low energy additions), or add electron where there is already one which costs  $U$  and there are  $1-x$  places to do that. So....

It must be that the spectral function (i.e. density of states for adding/removing electrons) looks like



This is quite different from band insulator case. (1) We added  $x$  holes but now we get  $2x$  low energy available states for electrons. (2) The high energy states get reduced in weight (upper Hubbard band) from 1 to  $1-x$ . A low-energy modification of removing electrons from lower Hubbard band had reduced the number of high-energy states at energy scale  $U$ ! If you like, we have moved  $x$  amount of spectral weight from high energy to low energy. And we get "twice as many" doped available electron states.

This analysis is correct for  $t \ll U$  (formally  $t/U=0$ ). For finite  $t$ , even more interesting things happen due to the fact that with hopping enabled we can make virtual transitions to doubly occupied states and so we get corrections of order  $t/U$  to these number of available states. These "dynamic" effects do something also interesting and counter-intuitive: they *increase* the weight  $2x$  to something even larger! We get "more doping" due to the dynamics, and apparently for cuprates this enhanced doping effect is seen in low-energy transport.

An interesting byproduct is that if one wants to produce a low-energy theory (i.e. a theory that works only in the low-energy manifold of states close to Fermi level), number conservation is not satisfied in the usual sense due to the dynamical effect, or at least one must redefine how one counts electrons/holes to get consistency: the number of active electronic degrees of freedom at low energy are not equal to those gotten from naive counting of electrons and doped holes.